Geometrie
Sätze am rechtwinkligen Dreieck – Der Satz des Pythagoras (II)

Pythagoras in Figuren und Körpern

1. Quadrat

a) $d^2 = 12^2 + 12^2$, $d = \sqrt{288} \approx 16{,}97\,cm$

b) $a^2 + a^2 = 10^2$
$2a^2 = 100$
$a^2 = 50$
$a = \sqrt{50} \approx 7{,}07$
$A = a^2 = 50\,cm^2$
$u = 4a = 4 \cdot 7{,}07 = 28{,}3\,cm$

c) $72 = a^2$
$d^2 = a^2 + a^2 = 144$
$d = \sqrt{144} = 12\,cm$

2. Rechteck

a) $d^2 = 45^2 + 22^2$, $d = \sqrt{2509} \approx 50{,}1\,cm$

b) $b^2 = 25^2 - 18^2$, $b = \sqrt{301} \approx 17{,}3\,cm$

3. gleichseitiges Dreieck

a) $h^2 = g^2 - \left(\frac{g}{2}\right)^2$, $h = \sqrt{60{,}75}$
$A = \left(\frac{g}{2}\right) \cdot h \approx 35{,}07\,cm^2$

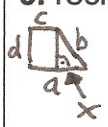
b) $a^2 = 20^2 + \left(\frac{a}{2}\right)^2$
$a^2 = 400 + \frac{a^2}{4}$
$\frac{3}{4}a^2 = 400$
$a^2 = \frac{1600}{3}$
$a = \sqrt{\frac{1600}{3}} \approx 23{,}09\,cm$

c) $h^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3}{4}a^2$
$h = \sqrt{\frac{3}{4}}\,a$, $A = h \cdot \frac{a}{2} =$
$\sqrt{\frac{3}{4}}\,a \cdot \frac{1}{2}a = 5$,
$a^2 = \sqrt{\frac{400}{3}}$, $a \approx 3{,}4\,cm$

4. gleichschenkliges Dreieck

$h^2 = 10^2 - \left(\frac{8}{2}\right)^2$, $h = \sqrt{84}$
$A = \left(\frac{8}{2}\right) \cdot h \approx 36{,}66\,cm^2$

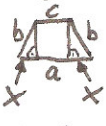
5. rechtwinkliges Trapez



$x = 35 - 28 = 7$
$b^2 = 7^2 + 30^2$, $b = \sqrt{949} \approx 30{,}81$
$u = a + b + c + d = 123{,}81\,cm$

$A = 28 \cdot 30 + \frac{7 \cdot 30}{2} = 945\,cm^2$

6. gleichschenkliges Trapez



$x = \frac{5{,}5 - 1{,}5}{2} = 2$
$h^2 = 3^2 - 2^2$, $h = \sqrt{5} \approx 2{,}24$
$A = \frac{1}{2}(5{,}5 + 1{,}5) \cdot h$
$= 7{,}84\,cm^2$

7. Raute

a) $\left(\frac{f}{2}\right)^2 = 68^2 - \left(\frac{64}{2}\right)^2$, $\frac{f}{2} = \sqrt{3600} = 60$, $f = 120$
$A = \frac{1}{2} \cdot 64 \cdot 120 = 3840\,cm^2$

b) $129{,}96 = \frac{1}{2} \cdot 13{,}8 \cdot f$, $f = 18{,}83$
$a^2 = \left(\frac{13{,}8}{2}\right)^2 + \left(\frac{18{,}83}{2}\right)^2$, $a \approx 11{,}68\,cm$

8. Parallelogramm

$h^2 = (\sqrt{10})^2 - x^2 = 10 - x^2$
$h^2 = 5^2 - (3+x)^2 = 25 - (9 + 6x + x^2)$
$= 25 - 9 - 6x - x^2 = 16 - 6x - x^2$
$\Rightarrow 10 - x^2 = 16 - 6x - x^2$, $-6x = -6$, $x = 1$
$h^2 = 10 - 1^2$, $h = \sqrt{9} = 3$
$A = a \cdot h = 3 \cdot 3 = 9\,cm^2$

9. Kegel

$h^2 = 40^2 - \left(\frac{20}{2}\right)^2$, $h = \sqrt{1500} \approx 38{,}73$
$V = \frac{1}{3}\pi \cdot 10^2 \cdot 38{,}73 = 4055{,}8\,cm^3$
$O = 10^2 \cdot \pi + 10 \cdot \pi \cdot 40 = 1570{,}8\,cm^2$

10. quadratische Pyramide

$d^2 = 10^2 + 10^2$, $d = \sqrt{200}$
$h^2 = 12^2 - \left(\frac{\sqrt{200}}{2}\right)^2$, $h = \sqrt{94} \approx 9{,}70$
$V = \frac{1}{3} \cdot 10^2 \cdot 9{,}70 = 323{,}33\,cm^3$
$h_s^2 = \left(\frac{10}{2}\right)^2 + 9{,}70^2$, $h_s = 10{,}91$
$O = 10^2 + 4 \cdot \left(\frac{1}{2} \cdot 10 \cdot 10{,}91\right) = 318{,}2\,cm^2$

11. Würfel

$d'^2 = 6^2 + 6^2$, $d' = \sqrt{72}$
$d^2 = d'^2 + 6^2$, $d = \sqrt{108} \approx 10{,}4\,cm$

Formel:
$d'^2 = 2a^2$, $d' = \sqrt{2a^2} = \sqrt{2} \cdot a$
$d^2 = d'^2 + a^2 = (\sqrt{2}a)^2 + a^2 = 2a^2 + a^2 = 3a^2$,
$d = \sqrt{3} \cdot a$

12. Quader

$d'^2 = 10^2 + 5^2$, $d' = \sqrt{125}$
$d^2 = d'^2 + 3^2$, $d = \sqrt{134} \approx 11{,}6\,cm$

Formel:
$d'^2 = a^2 + b^2$, $d' = \sqrt{a^2 + b^2}$
$d^2 = d'^2 + c^2 = (\sqrt{a^2 + b^2})^2 + c^2 = a^2 + b^2 + c^2$,
$d = \sqrt{a^2 + b^2 + c^2}$